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THE UNIVERSITY OF ALBERTA

AN AMPLIFIER FOR MEASUREMENT OF THE ELECTRICAL SIGNALS OF  
THE STOMACH FROM THE SURFACE OF THE BODY

by

FRED W. UNGER

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF MASTER OF SCIENCE

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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled An Amplifier for Measurement of the Electrical Signals of the Stomach from the Surface of the Body submitted by Fred W. Unger in partial fulfillment of the requirements for the degree of Master of Science.

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## ABSTRACT

The major portion of this thesis deals with the design of a solid state, low level amplifier suitable for measurement of the electrical activity of the stomach through electrodes placed on the surface of the body. While this is the main objective the amplifier developed is of such a nature that only minor changes are required to convert it for measurement of other low level physiological signals.

The nature of the electrical activity of the stomach is briefly introduced but no attempt is made to analyze the waves or correlate them with mechanical activity of the stomach.



## ACKNOWLEDGEMENTS

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## CHAPTER I

### INTRODUCTION

#### 1.1 General

Electrophysiological studies of the functions of internal organs such as the heart and brain have over the past years proved extremely valuable in the diagnosis of malfunction and disease. While these fields of electrocardiography and electroencephalography are widely studied and well developed the study of the electrical activity of the gastrointestinal tract or electrogastrography is not.

The basic reasons for this lack of development of electrogastrography seems to lie in the failure to find an effective method of tapping the bio-electric potentials from the gastrointestinal tract and in the subsequent filtering and amplification of these signals.<sup>(1)</sup>

Generally electrogastrograms are obtained by introduction of intragastric electrodes into the stomach cavity with the end of the tube making contact with the mucous membrane. Two major sources of error exist with this method.

Firstly, the contact with the mucous membrane is not always direct but is usually through a constantly changing layer of food and chyme. Thus it is very difficult to get an accurate amplitude measurement.

Secondly, the presence of the intragastric tube itself causes a reflex change in the activity of the vomiting center and of the center of the motor activity of the stomach which is functionally related to the



vomiting center. Thus it is also difficult to get an accurate frequency measurement.

Another method of approach is to record the electrical signals of the stomach from the surface of the body. <sup>(1,2)</sup> This approach presents many problems in both instrumentation and recording technique but does not have the two basic drawbacks of conventional electrogastrography.

## 1.2 Basic Mechanical and Electrical Activity of the Stomach

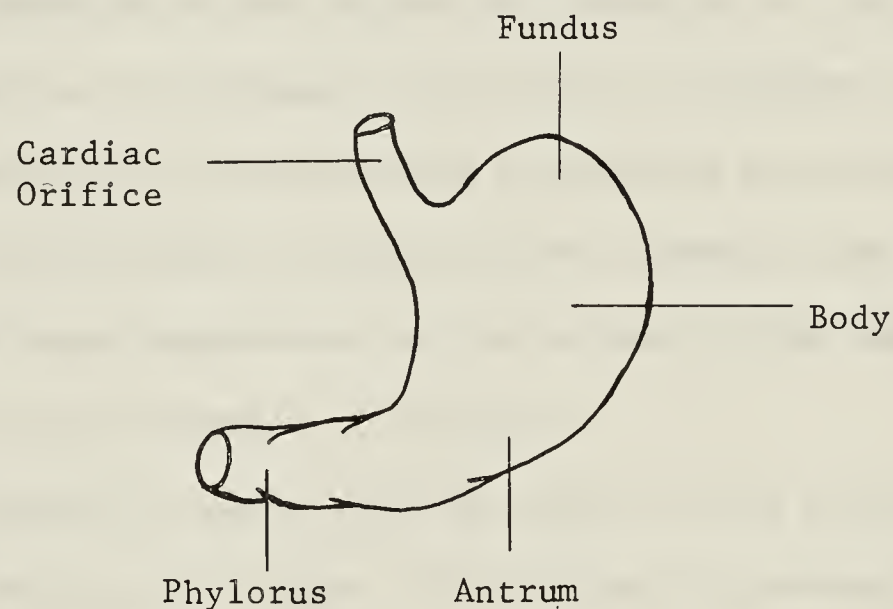


Fig. 1.1 Diagram of Parts of the Stomach

There are four main layers in the wall of the stomach: the mucosa, submucosa, external muscular layer and serosa. The only layer of interest here is the external muscular layer, whose function is to mix the contents of the stomach with the digestive juices and then move them along.



The muscular layer itself is composed of two very substantial sheets of smooth muscle, an inner circular sheet and an outer longitudinal sheet. Thus muscular contraction can either constrict the stomach or tend to shorten it or both. The resulting mechanical action which does occur can be described in terms of one basic component, a rhythmic activity which recurs normally at a rate of 3 per minute in man.<sup>(3)</sup> These peristaltic waves spread from the cardia throughout the antrum to the duodenum. The magnitude of the contractions accompanying the gastric rhythm may vary considerably depending on the degree of distention of the stomach, the chemical environment of the stomach, the release of hormones, etc.

Immediately preceding each peristaltic wave there occurs an electrical wave which varies in step with the rhythm of the peristaltic wave. This electrical wave spreads out to the surface of the body and can be picked up by proper placement of electrodes.

It should be noted that the signal at the electrodes is an aggregate signal from an entire organ. This signal in passing from the stomach to the electrodes is attenuated (not necessarily linearly) by layers of tissue. For these reasons the recorded signals may differ considerably in shape from that recorded by means of an electrode directly on the stomach.



## CHAPTER II

### INSTRUMENTATION REQUIREMENTS

#### 2.1 Electrodes

##### 2.1.1 General

A satisfactory "lead" for recording bio-electric potentials from the body surface depends on the following four factors:

1. Choice of the best electrode site.
2. Selection of the most suitable type of electrode.
3. Use of the most efficient attachment device.
4. Careful preparation of the skin.

The first of these factors is anatomical in nature and will be discussed later. The remaining three factors are instrumentation problems and will be discussed here.

##### 2.1.2 Type

Selection of the type of electrode to be used is generally determined by two factors:

1. Electrode area.
2. Inter-electrode resistance.

Considering these two factors in relation to the problem involved certain electrode characteristics are desirable.

Firstly, a large area electrode would probably not be satisfactory. This is due to the slow propagation of the waves in question. A wave would consume considerable time in merely crossing the electrode which would result in a broadening of the observed wave and a decreased resolution of events. Thus a small area electrode is desirable.



Secondly, the inter-electrode resistance should be as small as possible. This will reduce loading by the amplifier and also help to reduce stray pickup and noise.

#### 2.1.3 Attachment

The problem of attachment is not too important in this case since the portions of the body on which the electrodes are placed lend themselves readily to the most simple of attachment devices. However the method of attachment should be quick to use and also maintain a relatively constant pressure of the electrode against the skin.

#### 2.1.4 Skin Resistance

The skin consists of a membrane of two basic layers: a deep, living, metabolizing layer and a more superficial dead and dying layer of cells. Of these two layers the deep living layer accounts for only about 10% or 15% of the resistance of a given electrode. Thus the effect of this layer can be neglected. The outer layer should however be carefully prepared before electrode application. A method commonly used is to first cleanse the skin with alcohol or acetone to remove the oil present, and then abrade the skin slightly to remove some of the dead cells. Following this a conducting jelly or saline jelly is massaged into the skin before the electrodes are applied.

#### 2.1.5 Electrodes Chosen

The electrodes chosen were the "Beckman Biopotential Skin Electrodes". These electrodes are small in physical size and very easy to attach. The silver-silver chloride pellet which serves as the elec-



trode element is set in a small plastic casing, one side of which is perforated by four holes. Thus the electrode element is protected from the subjects skin and electrical contact is made through the electrolyte gel which is placed in the holes. With this method motion artifacts due to rubbing of the electrode element against the skin are almost completely eliminated.

The electrodes are attached by means of a two-sided adhesive collar. One side of the adhesive collar sticks to the subject while the other side sticks to the plastic case. When properly applied the inter-electrode resistance is of the order of 5 to 10 kilohms.

## 2.2 Amplifier Requirements

### 2.2.1 General

The bio-electric potentials of the stomach as measured on the surface of the body are of the order of 200 to 300 microvolts peak to peak, with a frequency of 2 to 4 per minute. Along with these signals at the skin surface there are potentials from the other organs of the body as well as the galvanic skin response itself. The frequency spectrum of these potentials is quite wide.

Considering the above signal conditions, specifications can be set up for the amplifier.

### 2.2.2 Frequency Response

Since no rest potential is to be recorded, and slow D.C. skin potential variations along with other very low frequency signals serve only to complicate the recordings, suppression of signals below approximately 1 per minute is desirable. Also the suppression of higher fre-



quency signals (above 5 or 6 per minute) would help to keep the recordings clean and undistorted.

#### 2.2.3 Gain

The gain of the amplifier should be fairly large so that any following amplifiers used (as in recorders) need not be of the ultra low drift type. The gain should be stable and well defined to allow accurate level measurements.

#### 2.2.4 Noise

The total noise voltage from the source and amplifier should be less than 8 microvolts peak to peak referred to the input. This will keep the error in a 200 microvolt peak to peak signal equal to or less than 4%.

#### 2.2.5 Input Resistance

The "loading loss" due to the amplifier should be kept at less than 1%. Since the electrode resistances are of the order of 5 to 10 kilohms an amplifier input resistance of at least 1 megohm is required.

#### 2.2.6 Differential

Due to the close proximity of the stomach and the heart considerable interference by heart action currents may occur even though they are of much higher frequency and lie outside the pass band. For this reason the amplifier should be differential in form and have a high common mode rejection ratio. Then if the electrodes are placed on an isopotential heart line very little interference will occur. Differential action of the amplifier will also reduce greatly the effect of 60 cps pickup.



### 2.3 Proposed System

Considering the preceding requirements the following basic system was proposed: two stages of amplification for a total gain of 1000; the input stage to be differential and the second stage single ended; the required high input impedance and low frequency shaping to be obtained in the input stage by bootstrapping. The high frequency shaping to be obtained in the second stage, which would act as a low pass active filter.



## CHAPTER III

### AMPLIFIER DESIGN

#### 3.1 Input Stage

##### 3.1.1 General

The design of the input stage can be considered in two steps:

1. Design of the bootstrapped bias network.
2. Design of the amplifier.

While these two steps are not separate they are a logical division point since specification of the bootstrapped bias components then determines the properties required by the amplifier for best system performance.

##### 3.1.2 Bootstrapped Bias Network

The bootstrapped bias network serves two purposes:

1. Raising of the input impedance.
2. Low frequency response shaping.

Edwards <sup>(6)</sup> has shown that the bootstrapped network of Fig. 3.1 can be represented as an amplifier with feedback as in Fig. 3.2.



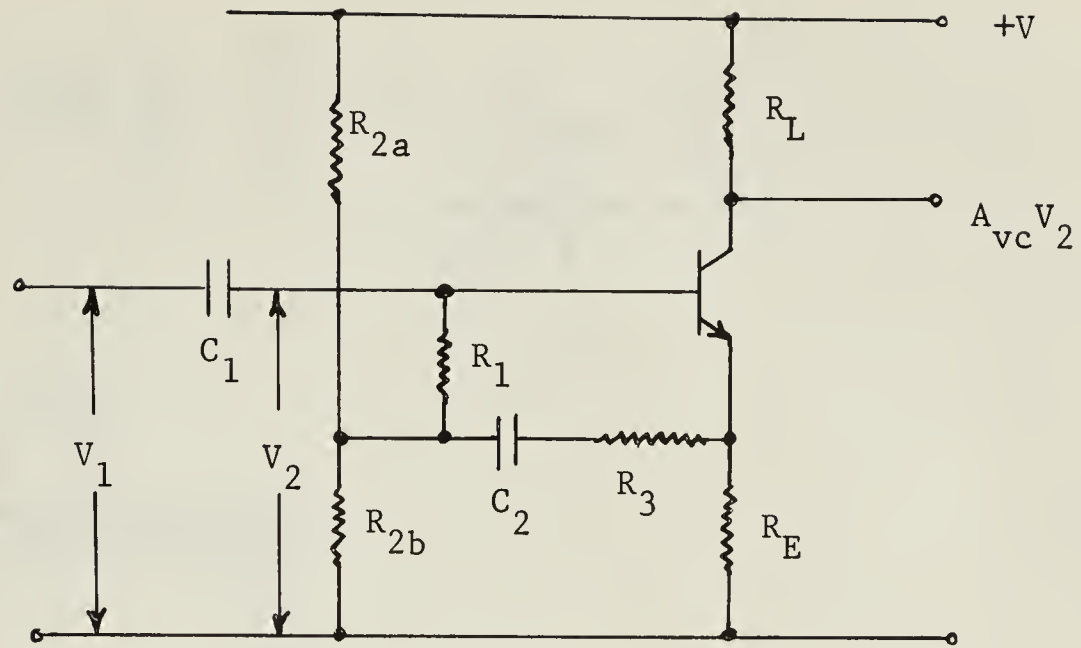


Fig. 3.1 Bootstrapped Bias Stage

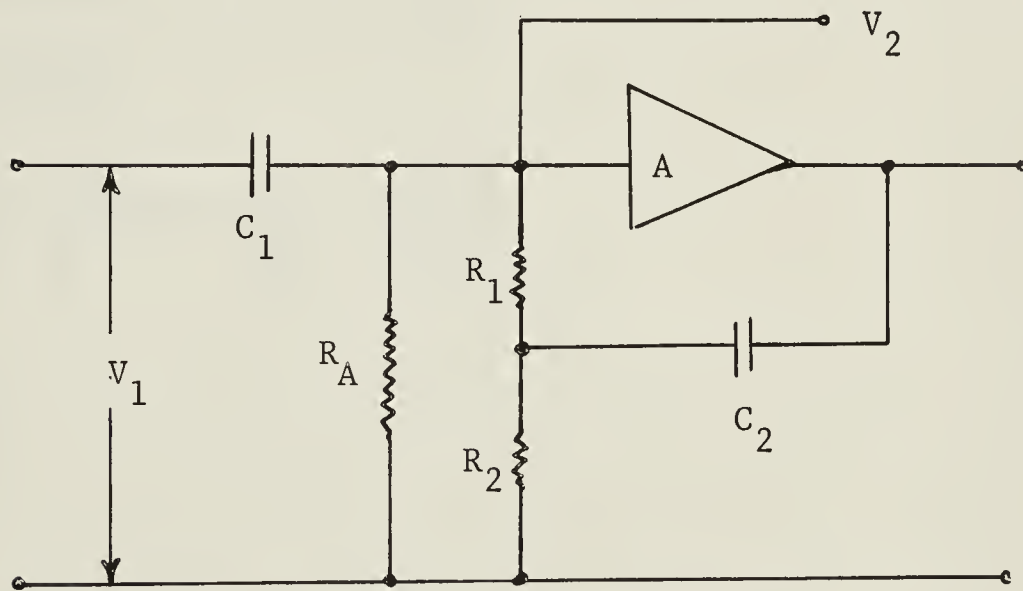


Fig. 3.2 Equivalent Circuit of Bootstrapped Bias

The transfer function of this system is:

$$H[P] = \frac{V_2}{V_1} = \frac{P^2 + 2\zeta_1 P}{P^2 + 2\zeta P + 1}$$

Where  $P = T_o s = \frac{s}{\omega_o}$

$\omega_o$  being the normalized frequency.

$$T_o = \sqrt{\alpha(1-\alpha) \frac{R_A R_C^2}{R R_A + R_C} C_1 C_2}$$

$$\alpha = \frac{R_1}{R_C} \quad R_C = R_1 + R_2$$



$$\zeta = \frac{1}{2} \sqrt{\frac{1-\alpha}{\alpha} \frac{C_2}{C_1}} \left[ \frac{(1-A) + \frac{C_1}{(1-\alpha) C_2} + \frac{\alpha R_C}{R_A}}{\sqrt{1 + \frac{R}{R_A}}} \right]$$

$$\zeta_1 = \frac{\sqrt{\frac{C_1}{C_2}}}{2 \sqrt{\alpha(1-\alpha) \left(1 + \frac{R_C}{R_A}\right)}}$$

If  $R_1=R_2$ , that is  $\alpha=\frac{1}{2}$  and  $R_A$  is very much greater than  $R_C$  then the equations reduce to the following.

$$T_o \approx \frac{1}{2} R_C \sqrt{C_1 C_2}$$

$$\zeta \approx \frac{1}{2} \sqrt{\frac{C_2}{C_1}} \left[ (1-A) + \frac{2C_1}{C_2} + \frac{\alpha R_C}{R_A} \right]$$

$$\zeta_1 \approx \sqrt{\frac{C_1}{C_2}}$$

### 3.1.3 Converting to a Third Order System

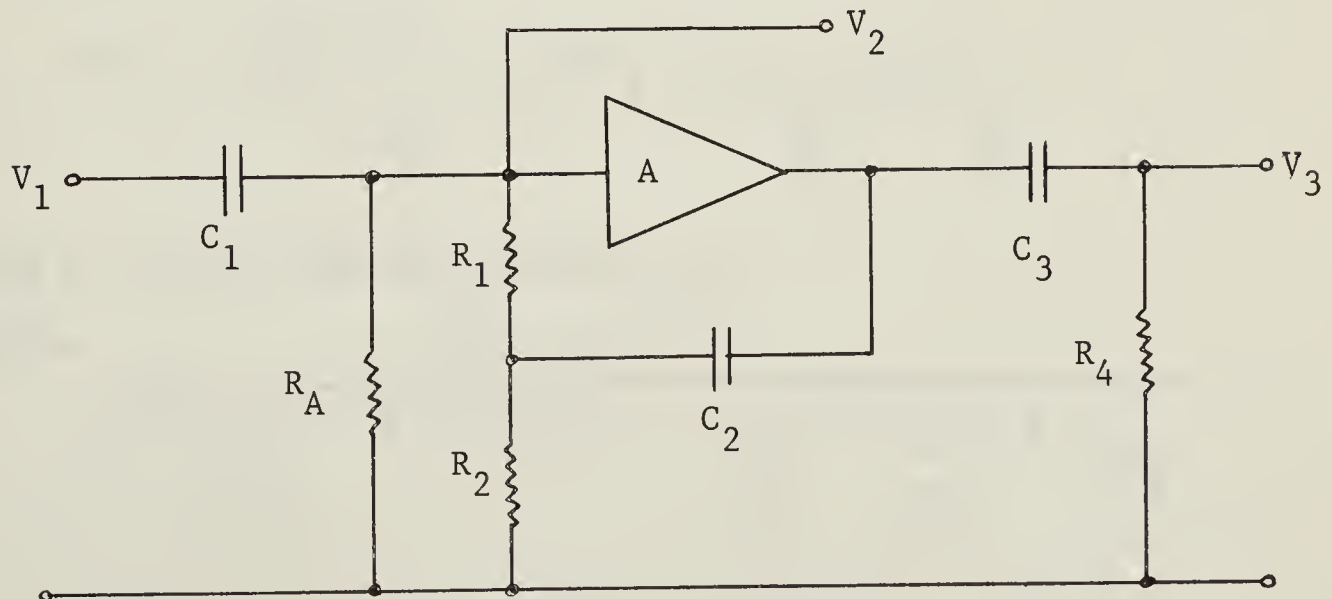


Fig.3.3 Conversion of Bootstrapped Input to a Third Order System



The transfer function of the bootstrapping is:

$$H_1[P] = \frac{P^2 + 2\zeta_1 P}{P^2 + 2\zeta P + 1}$$

For the added RC network the transfer function is:

$$H_2(s) = \frac{T_1 s}{1 + T_1 s} \quad \text{Where } T_1 = R_4 C_3$$

$$\text{Since } s = \frac{P}{T_o}$$

$$H_2[P] = \frac{\frac{T_1}{T_o} P}{1 + \frac{T_1}{T_o} P}$$

The cascaded transfer function is then:

$$H[P] = \frac{P^2 + 2\zeta_1 P}{P^2 + 2\zeta P + 1} \cdot \frac{\frac{T_1}{T_o} P}{1 + \frac{T_1}{T_o} P}$$

Since the roots of the RC transfer function are always real, the bootstrapping will determine the damping present in the system.

The maximally flat conditions can be found from the transfer function:

$$H[P] = \frac{P^3 + 2\zeta_1 P^2}{P^3 + \left(\frac{T_o}{T_1} + 2\zeta\right) P^2 + \left(1 + 2\frac{T_o}{T_1} \zeta\right) P + \frac{T_o}{T_1}}$$

Letting  $P = ju$  the amplitude response is:

$$|H[ju]|^2 = \frac{u^6 + 4\zeta_1^2 u^4}{u^6 + \left(\frac{T_o^2}{T_1^2} - 2 + 4\zeta^2\right) u^4 + \left(1 + 4\frac{T_o^2}{T_1^2} \zeta^2 - 2\frac{T_o^2}{T_1^2}\right) u^2 + \frac{T_o^2}{T_1^2}}$$



The conditions for maximal flatness are thus:

$$\begin{aligned} 1. \quad & 4\zeta^2 - 4\zeta_1^2 - 2 + \frac{T_o^2}{T_1^2} = 0 \\ 2. \quad & 1 - 2 \frac{T_o^2}{T_1^2} (1 - 2\zeta^2) = 0 \end{aligned}$$

From the second condition it can be seen that  $\zeta$  must be less than .707 for the equality to hold. Thus to achieve maximal flatness the system must be underdamped.

From the equations describing the maximally flat conditions and the equations given in section 3.1.2 the needed component values can be found. Many approaches to the determination of these components can be taken at this point. In this case the choosing of  $C_1$  and  $C_2$  is one of the best, since the values of these capacitors affect noise performance, input impedance and roll-off.

The impedance of the bootstrapped resistance  $R_1$  as seen at the input is approximately,

$$R_B = \frac{\alpha R_C}{1-A}$$

Where, referring to Fig. 3.1;

$$A = \frac{R_2}{R_2 + R_3} A_{VE} \quad R_2 = R_{2a} // R_{2b}$$

The input resistance to the system is then:

$$R_{IN} = R_B // R_A$$

For a given cut-off frequency ( $T_o$ ), keeping the product  $C_1 C_2$  small results in a larger  $R_C$  and thus in a larger input resistance.



It can be shown that to obtain the best attenuating effects below the -3db point  $\zeta_1$  should be kept as small as possible. (See Appendix II) Since  $\zeta_1 \cong \sqrt{\frac{C_1}{C_2}}$  the ratio of  $C_1$  to  $C_2$  should be kept small.

The equation for the equivalent noise voltage referred to the input (Johnson noise and amplifier noise) is: <sup>(6)</sup>

$$V_{\text{RMS}}^2 = 4KT\Delta f \left[ F_o R_g + R_n R_g^2 \left( \frac{1}{R_g} - \frac{1}{R_o} \right)^2 + \frac{R_n}{\omega^2 C_1^2 R_o^2} \right]$$

From this equation it can be seen that the optimum value of  $C_1$ , regardless of the operating current of the input transistor, is infinite.

Clearly a compromise must be made in the magnitudes of  $C_1$  and  $C_2$ , since one condition calls for small values while another calls for large values. One other problem must also be considered; that of leakage current in large value electrolytic capacitors. The leakage of  $C_1$  drives directly into the amplifier and can cause considerable amount of low frequency noise at the output. The effect of the leakage of  $C_2$  is not nearly as great, but it can still cause considerable noise if  $C_2$  is very large. Thus considering leakage current  $C_1$  and  $C_2$  should not be too large.

Taking into account the above conditions, values of  $C_1=60\text{uf}$  and  $C_2 = 300\text{uf}$  were chosen.

These values appear to give satisfactory results with respect to all the above mentioned factors. The product  $C_1 C_2$  is small enough so that the required input impedance can be obtained and yet  $C_1$  is large enough to give good noise performance. The resulting roll-off begins



at 20db per decade and then quickly changes to 40db per decade which is satisfactory. The use of good quality tantalum capacitors of this size produce only minor leakage problems. Also the design of the amplifier circuit such as that as small a D.C. voltage as possible appears across the capacitors helps to minimize leakage.

### 3.1.4 Component Values

With  $\alpha = \frac{1}{2}$  and  $R_A \gg R_C$

$$\zeta_1 \approx \sqrt{\frac{C_1}{C_2}} = .447$$

Applying the maximally flat conditions;

$$\frac{T_o^2}{T_1^2} = 1.475$$

and  $\zeta = .575$

For a -3db point at .005 cps,

$$\frac{\frac{T_o^2}{T_1^2}}{u^6 + 4\zeta_1^2 u^4} = 1$$

$T_o = 30.6$  sec. and  $T_1 = 25.2$  sec.

$$R_C \approx \frac{2T_o}{\sqrt{C_1 C_2}}$$

or  $R_1 = R_2 = 228K$

$$\zeta \approx \frac{1}{2} \sqrt{\frac{C_2}{C_1} \left( (1-A) + \frac{2C_1}{C_2} \right)}$$

$(1-A) = .115$

Thus with  $R_A \gg R_B$  the input impedance to the system will be approximately,

$$R_{in} = \frac{\alpha R_C}{1-A} = 2 \text{ megohms}$$



### 3.1.5 Amplifier Circuit

Having chosen a value of  $C_1=60\mu\text{f}$  for the input capacitor the optimum operating current of the input transistor can be found from the noise equation. The value found is approximately one micro-amp. Also in designing the bootstrapped bias it was assumed that the input impedance to the amplifier was much greater than  $R_1+R_2=458\text{K}$ . Thus the input impedance should be of the order of 20 megohms. To obtain the high input impedance and low operating current level of the input transistor it was decided to use an emitter follower driving a common emitter stage for each side of the differential amplifier.

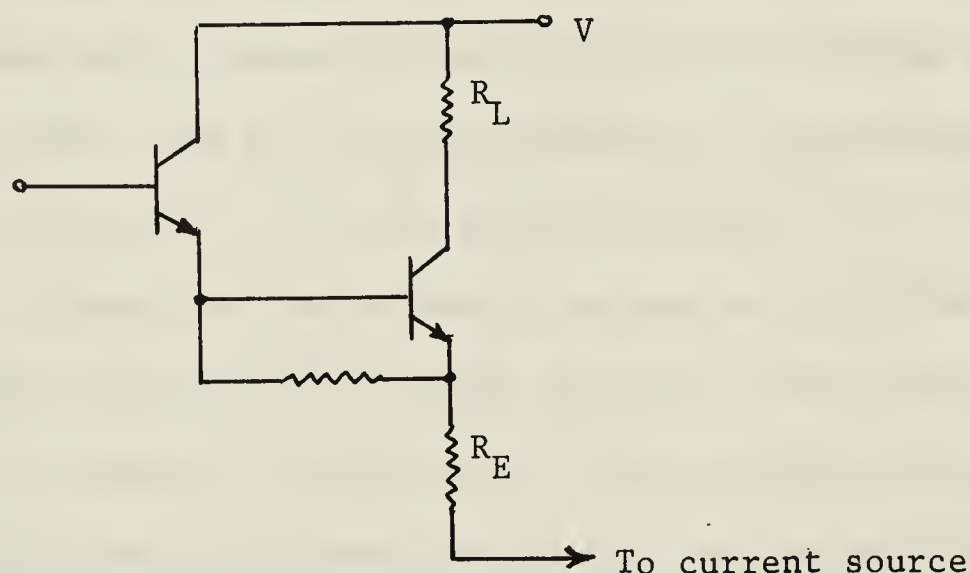


Fig. 3.4 Basic Circuit of One Side of Differential Amplifier

In the above common emitter stage the major departure of the gain from the ideal value  $G=-\frac{R_L}{R_E}$ , is due to the term  $\frac{h_{ib}}{R_E}$  since  $h_{ib}$  is inversely proportional to the emitter current. This source of non-linearity can be greatly reduced by replacing the individual transistors by two transistor compound blocks.<sup>(7)</sup> (See Appendix I)



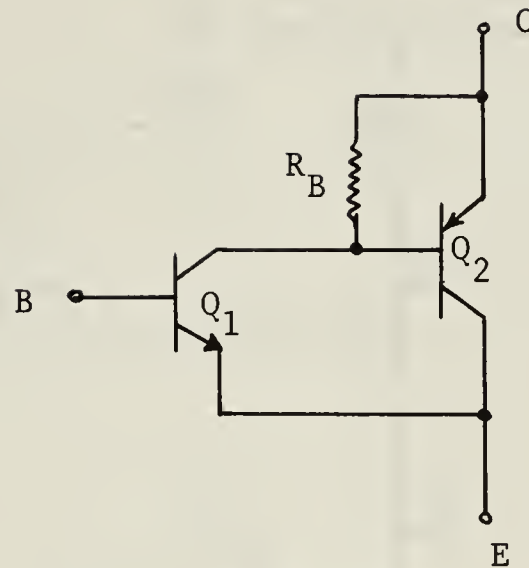


Fig. 3.5 Two Transistor Compound

The above compound can be considered as a single NPN transistor with a current gain approximately equal to the product of the current gains of the individual units. The  $h_{ib}$  for the compound is approximately  $h_{ib}$  of  $Q_1$  divided by  $h_{fe}$  of  $Q_2$ . If the collector current of  $Q_1$  is made several times larger than the collector current of  $Q_2$  divided by its current gain then the collector current of  $Q_1$  will vary only slightly over the operating range of the compound. Thus not only is  $h_{ib}$  reduced but the variations in  $h_{ib}$  are also greatly reduced which gives a stable and well defined voltage gain. By using a two transistor compound for the emitter follower the input impedance to the amplifier becomes limited by  $h_{ob}$  of the input transistor. For the above reasons it was decided to use compound blocks in place of the individual transistors in Fig. 3.4.

The completed amplifier circuit is shown in Fig. 3.6. The bootstrapped bias is also shown.



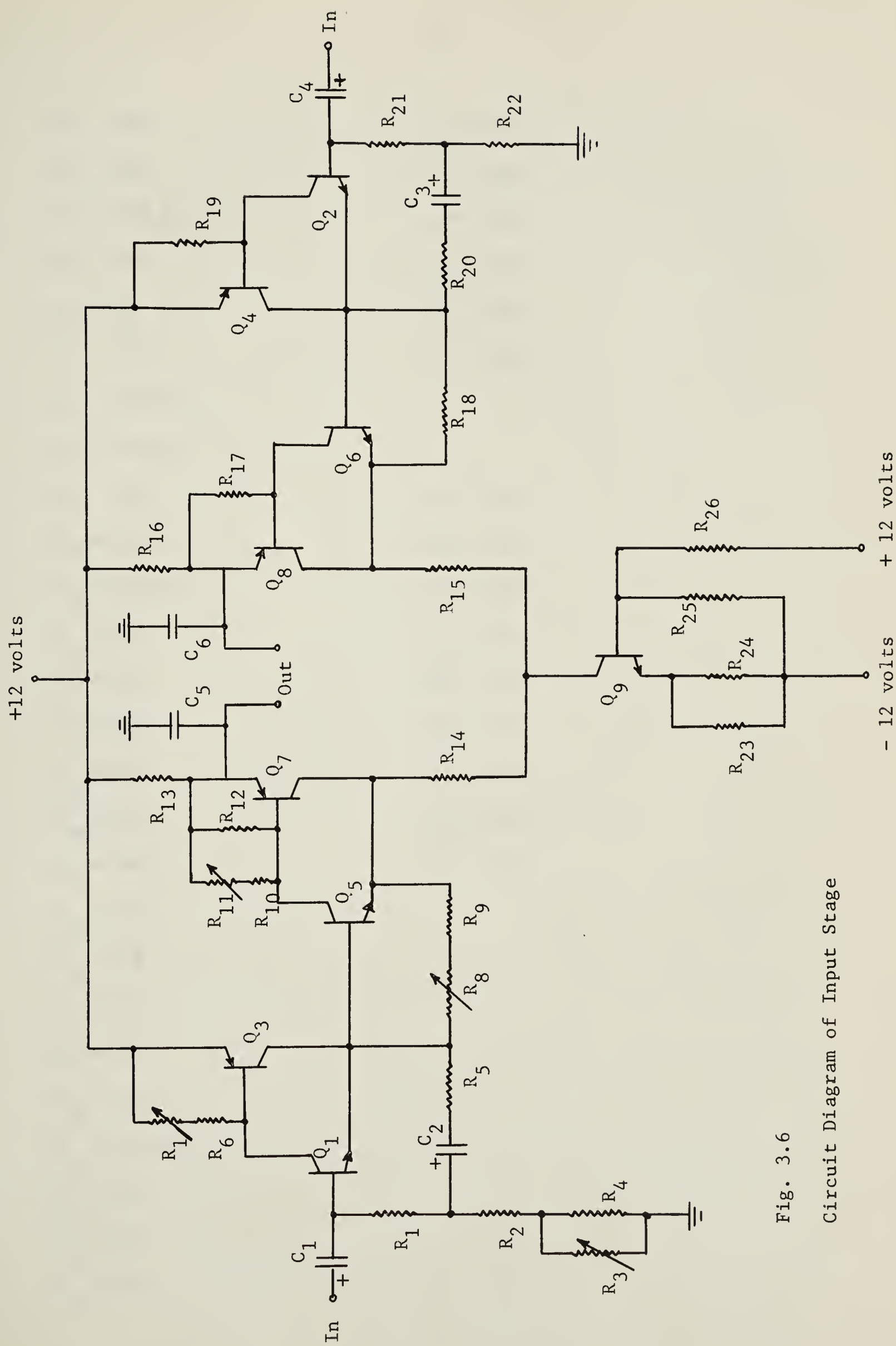


Fig. 3.6

Circuit Diagram of Input Stage



$R_1 = 220K$	$C_1 = 60\mu f$
$R_2 = 180K$	$C_2 = 300\mu f$
$R_3 = 100K \text{ Pot.}$	$C_3 = 300\mu f$
$R_4 = 220K$	$C_4 = 60\mu f$
$R_5 = 27K$	$C_5 = .68\mu f$
$R_6 = 470K$	$C_6 = .68\mu f$
$R_7 = 200K \text{ Pot.}$	
$R_8 = 50K \text{ Pot.}$	
$R_9 = 33K$	$Q_1 - T1415$
$R_{10} = 8.2K$	$Q_2 - T1415$
$R_{11} = 10K \text{ Pot.}$	$Q_3 - 2N3251$
$R_{12} = 15K$	$Q_4 - 2N3251$
$R_{13} = 10K$	$Q_5 - T1415$
$R_{14} = 100 \Omega$	$Q_6 - T1415$
$R_{15} = 100 \Omega$	$Q_7 - 2N3251$
$R_{16} = 10K$	$Q_8 - 2N3251$
$R_{17} = 6.8K$	$Q_9 - T1415$
$R_{18} = 56K$	
$R_{19} = 560K$	
$R_{20} = 27K$	
$R_{21} = 220K$	
$R_{22} = 220K$	
$R_{23} = 25K \text{ Pot.}$	
$R_{24} = 5.6K$	
$R_{25} = 18K$	
$R_{26} = 33K$	



Each side of the differential amplifier consists of an emitter follower driving a common emitter stage. Both the emitter follower and common emitter stages are made up of two transistor compounds.

$Q_9$  serves as a high impedance current source for the emitter circuit.

The operating currents of the transistors are as follows:

$Q_1$  and  $Q_2$  ... 1 ua

$Q_3$  and  $Q_4$  ... 10 ua

$Q_5$  and  $Q_6$  ... 100 ua

$Q_7$  and  $Q_8$  ... 1 ma

$Q_9$  ... 2.22 ma

Under no signal conditions the bases of  $Q_1$  and  $Q_2$  are at approximately zero volts D.C. and the output is at +2 volts D.C. The D.C. output can be adjusted by changing the setting of potentiometer  $R_{23}$ . This allows the setting of the correct bias voltage for the second stage.

The RC network which converts the system to third order was not added to the output of this stage. Instead it was placed at the output of the second stage. The reasons for this are discussed in section 3.2.3.

$C_5$  and  $C_6$  were placed across the output to reduce the bandwidth and improve the common mode rejection ratio at high frequencies. With the values chosen the high frequency cut-off of the amplifier is approximately 23 cps. Thus  $C_5$  and  $C_6$  will in no way interfere with the frequency shaping of the following low pass filter stage.



### 3.1.6 Balancing the Amplifier

Balance of the differential amplifier is obtained by varying potentiometers  $R_3$ ,  $R_7$ ,  $R_8$  and  $R_{11}$ . With the base of  $Q_1$  shorted to the base of  $Q_2$  and the load resistors matched, zero differential output voltage will be obtained from the amplifier if the collector currents of the corresponding compound transistors are exactly equal. The emitter currents will then be very nearly equal since the corresponding compound transistors have very large, nearly equal current gains. Thus the voltage drops across the emitter resistors will be very nearly equal. This is possible only if the base-emitter voltages of the corresponding compound transistors are equal for equal collector currents. Since the base-emitter voltage of modern silicon transistors is a quite predictable parameter only small variations usually occur. The base-emitter voltages of the corresponding transistors used were matched to within 2 milli-volts. The remaining difference present was removed by adjustment of  $R_7$  and  $R_{11}$ . Referring to Fig. 3.5, if  $R_B$  is varied while the compound collector current is held constant, the collector current of  $Q_1$  will be varied, and thus the base-emitter voltage will change. With close initial matching of base-emitter voltage only a small variation in the collector current is required.

The complete balancing procedure for the amplifier is as follows:

1. Place a short between the base of  $Q_1$  and the base of  $Q_2$ .
2. Adjust  $R_7$  for zero differential voltage between the base of  $Q_5$  and the base of  $Q_6$ .



3. Place a short between the base of  $Q_5$  and the base of  $Q_6$ .
4. Adjust  $R_{11}$  for zero differential voltage between the emitter of  $Q_5$  and the emitter of  $Q_6$ .
5. Remove the short between the base of  $Q_5$  and the base of  $Q_6$ .
6. Adjust  $R_8$  for zero differential voltage at the output.
7. Remove the short between the base of  $Q_1$  and the base of  $Q_2$ .
8. Adjust  $R_3$  for zero differential voltage at the output.

### 3.1.7 Input Impedance

In calculating the input impedance to the common emitter stage the best approach is to combine the current shunting resistor ( $R_{18}$ ) with the input transistor ( $Q_6$ ) of the compound. The major effect on the transistor is reduction of  $h_{fe}$ . (See Appendix I - effect of  $R_A$  on  $Q_2$ .) The input impedance seen looking into the common emitter stage is then given by the expression:

$$Z_{iE} = (1+g)(h_{fe}^* Z_E // \frac{1}{h_{ob}^* (1+G)})$$

Where

$$G = \frac{R_{14}}{R_{15}}$$

$$g = \frac{h_{ib}^*}{Z_E} + Gh_{ob.E}^* + G(h_{rb}^* + h_{ib}^* h_{ob}^*)$$

$$Z_E = R_{15}$$

To evaluate this expression the values of the  $h^*$  parameters must be found. Using the approximate equations from Appendix I and the values of the  $h$  parameters for the individual transistors:



$$h'_{fe_1} = 150$$

$$h'_{fe_2} = 111$$

$$h^*_{ib} = 2.34\Omega$$

$$h^*_{fe} = 16,600$$

$$h^*_{ob} = .4 \times 10^{-7}$$

$$h^*_{rb} = 1.97 \times 10^{-7}$$

then  $Z_{iE} = 218K$

This impedance appears in parallel with the loading by the bias network to form the emitter load for the input compound. At midband the load of the bias network is:  $Z_B = 247K$ . Thus  $Z_E = 218K // 247K = 116K$ .

The input impedance seen looking in at the base of  $Q_2$  is given by the expression:

$$Z_{iC} = h^*_{fe} Z_E // \frac{1}{h^*_{ob}}$$

Where  $Z_E = 116K$  and  $h^*_{fe}$ ,  $h^*_{ob}$  are the  $h^*$  parameters of the input compound. For the input compound:

$$h'_{fe_2} = 66$$

$$h^*_{ib} = 394\Omega$$

$$h^*_{fe} = 5270$$

$$h^*_{ob} = .4 \times 10^{-7}$$

$$h^*_{rb} = 11.6 \times 10^{-4}$$

Thus  $Z_{iC} = 24$  megohms.



The input impedance to the entire system is then:

$$R_{in} = \frac{\alpha R_C}{1-A} // 24M = 1.85 \text{ megohms}$$

### 3.1.8 Gain

The gain from the base to the emitter of the input compound is given by the expression:

$$A_{vC} = \frac{1-h_{rb}^*}{1+h_{ib}^* \frac{z_E}{z_E}}$$

Using the values for  $h_{rb}^*$ ,  $h_{ib}^*$  and  $z_E$  as found in the preceding section:

$$A_{vC} = .9946$$

The gain of the common emitter stage is given by the expression:

$$A_{vE} = - \frac{G}{1+g}$$

Where  $G = \frac{R_{14}}{R_{15}}$  and  $g = \frac{h_{ib}^*}{z_E} + Gh_{ob}^* z_E + G(h_{rb}^* + h_{ib}^* h_{ob}^*)$

Using the values of the  $h^*$  parameters from the preceding section:

$$A_{vE} = -95.8$$

The overall differential gain of the input stage is then:

$$A_{vD} = A_{vC} A_{vE} = -95$$

Since the amplifier will be operated with single ended output the gain becomes:

$$A_v = - \frac{95}{2} = -47.5$$



### 3.1.9 Common Mode Rejection

The output impedance of the current source is given by the expression:

$$z_{oE} = \frac{1}{h_{ob} + (h_{ob} + h_{rb}) \left( \frac{h_{fe}}{1 + h_{fe} (h_{ob} z_E + \frac{h_{ib} + z_E}{z_g})} \right)}$$

$$z_{oE} = 1.11 \text{ megohms}$$

The common mode gain is then:

$$A_{cm} = \frac{5K}{1.11M} = .0045$$

The common mode rejection ratio for single ended output at midband is then:

$$CMR = \frac{47.5}{.0045} = 10,550 = 80.5 \text{ db}$$

## 3.2 Second Stage

### 3.2.1 General

The design of the second stage can also be considered in two steps:

1. Design of RC sections within the feedback loop for the required frequency shaping. (See Appendix II)
2. Design of the amplifier.

Again these two steps are not entirely separate since the value of  $R_1$  sets an acceptable level on the input impedance of the amplifier and also determines the input transistors operating current for best noise performance.



### 3.2.2 Design of RC Sections

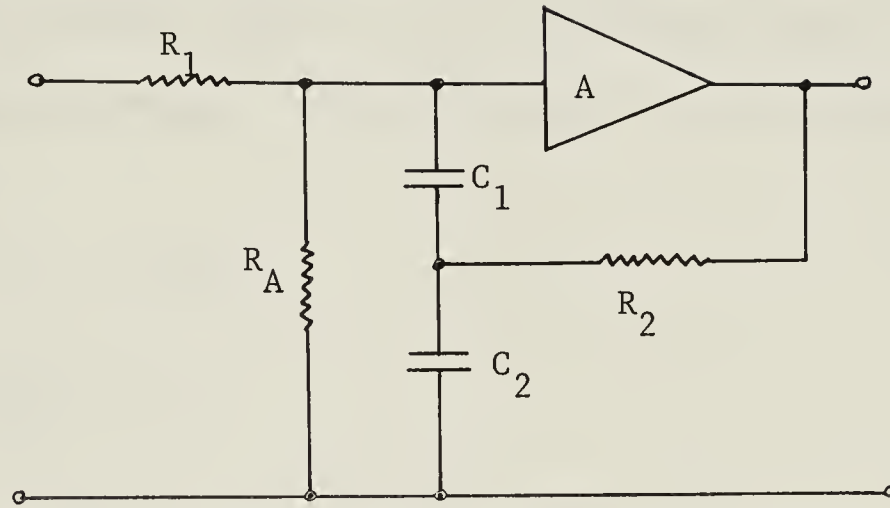


Fig. 3.7 Low Pass Active Filter

The requirements which must be met by the input arrangement are; maximally flat with a 3db down point at .2 cps and a roll-off of 40db per decade for an octave or more. From the maximally flat condition  $\gamma=0$ , and for a roll-off of 40db per decade for an octave  $\zeta_1$  must be approximately .25.

Thus choosing  $\zeta_1 = .25$

$$\gamma = 0 = 2\zeta^2 - 2\zeta_1^2 - 1$$

Then  $\zeta=.75$ . Thus the system is slightly underdamped.

$|H[ju]|$  must be down 3db at .2 cps.

$$|H[ju]|^2 = \frac{1}{1 + \frac{u^4}{4\zeta_1^2 u^2 + 1}}$$

$$\text{then } u^4 - 4\zeta_1^2 u^2 - 1 = 0$$

Since  $u = T_o \omega$ ,  $T_o = .85$  sec.



From this point many approaches<sup>e</sup> can be taken to the determination of the proper component values. One of the easiest ways is the choosing of the capacitor values. This allows the use of standard sized capacitors.

Letting  $C_1 = C_2 = 100\mu\text{f}$

$$\text{Then } \zeta_1 \cong \sqrt{\frac{R_2}{R_1}} = .25$$

$$T_o^2 \cong R_1 R_2 C_1 C_2 = .722 = .063 \times 10^{-8} R_1^2$$

and  $R_1 = 33.8\text{K}$

$$R_2 = 2.14\text{K}$$

The above component values are entirely acceptable. Although  $C_1$  and  $C_2$  are large value electrolytic capacitors, leakage current problems are not as great in this stage as in the input stage. Also using smaller values would result in a larger  $R_1$  which would increase thermal stability and noise problems.

### 3.2.3 Amplifier Circuit

The amplifier circuit used in the second stage is the same as that used for one side of the input stage, except that the emitter follower input uses only a single transistor instead of a compound.

The complete second stage is shown in Fig. 3.8.



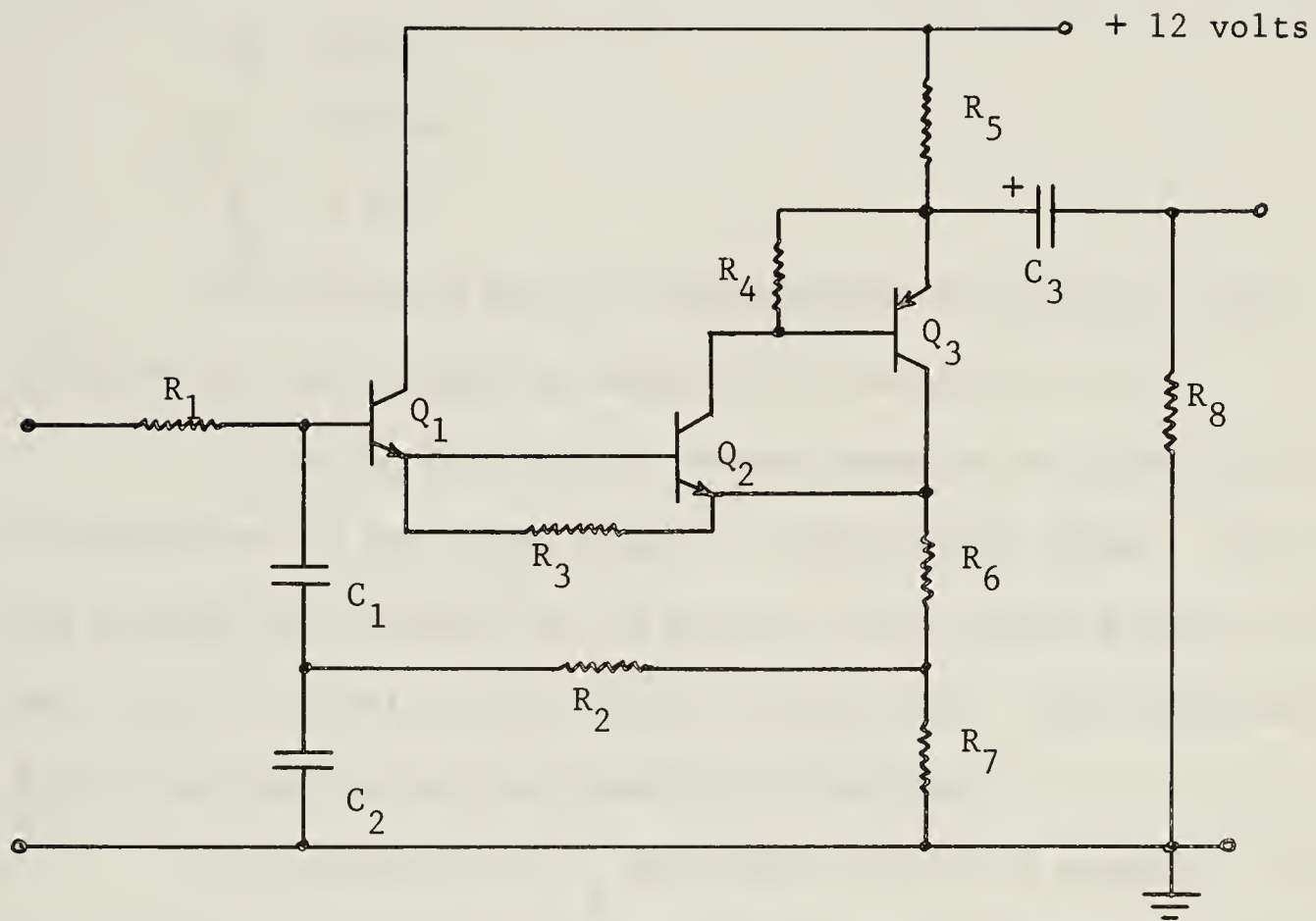


Fig. 3.8 Circuit Diagram of Second Stage

$$R_1 = 22K$$

$$R_2 = 2.2K$$

$$R_3 = 68K$$

$$R_4 = 6.8K$$

$$R_5 = 8.2K$$

$$R_6 = 68\Omega$$

$$R_7 = 270\Omega$$

$$R_8 = 5.6M$$

$$C_1 = 100\mu f$$

$$C_2 = 100\mu f$$

$$C_3 = 10\mu f$$

$$Q_1 - T1415$$

$$Q_2 - T1415$$

$$Q_3 - 2N3251$$



The operating currents of the transistors are as follows:

$$Q_1 - 10 \text{ ua}$$

$$Q_2 - 100 \text{ ua}$$

$$Q_3 - 1 \text{ ma}$$

The required bias is approximately +1.6 volts. This is supplied by the input stage as mentioned in section 3.1.5.

$C_3$  and  $R_8$  form the RC network required to convert the bootstrapped bias of the input stage to a third order system. The reason for placing this network at the output of the second stage is that it sets the D.C. level at the output to zero volts. Thus when used with a D.C. recorder no zero suppression is required.

In section 3.1.4  $T_1$  was found to be 25.2 seconds. Thus  $C_3(R_8//R_R) = 25.2$  seconds. Where  $R_R$  is the input impedance to the recorder. (For the recorder used  $R_R = 5$  megohms.)

#### 3.2.4 Input Impedance

The calculation of the input impedance follows closely with that given in section 3.1.7.

Looking into the common emitter stage:

$$z_{iE} = (1+g) \left( h_{fe}^* z_E // \frac{1}{h_{ob}^* (1+G)} \right)$$

Where  $G = \frac{R_5}{R_6 + R_7}$

$$g = \frac{h_{ib}^*}{z_E} + G h_{ob}^* z_E + G(h_{rb}^* + h_{ib}^* h_{ob}^*)$$

$$z_E = R_6 + R_7$$



Again the effect of the current shunting resistor ( $R_3$ ) on the current gain of  $Q_2$  must be taken into account.

$$h'_{fe1} = 142$$

$$h'_{fe2} = 103$$

$$h_{ib}^* = 2.52\Omega$$

$$h_{fe}^* = 14,600$$

$$h_{ob}^* = .4 \times 10^{-7}$$

$$h_{rb}^* = 1.9 \times 10^{-4}$$

Then  $z_{iE} = 840K$

This forms the emitter load for  $Q_1$ . The input impedance seen looking into the base of  $Q_1$  is given by the expression:

$$z_{iC} = h_{fe} z_E // \frac{1}{h_{ob}}$$

Where  $z_E = 840K$

Thus  $z_{iC} = 18 \text{ Meg.} = R_A$  and  $R_A \gg R_1$

### 3.2.5 Gain

The gain of the common emitter stage is:

$$A_{vE} = - \frac{G}{1+g}$$

Where  $G = \frac{R_5}{R_6 + R_7}$

$$g = \frac{h_{ib}^*}{z_E} + G h_{ob}^* z_E + G (h_{rb}^* + h_{ib}^* h_{ob}^*)$$

$$z_E = R_6 + R_7$$

Using the values of the  $h^*$  parameters from the preceding section:

$$A_{vE} = -22.8$$



The gain from the base to the emitter of  $Q_1$  is:

$$A_{vC} = \frac{1 - h_{rb}}{1 - \frac{h_{ib}}{Z_E}}$$

Where  $Z_E = 840K$

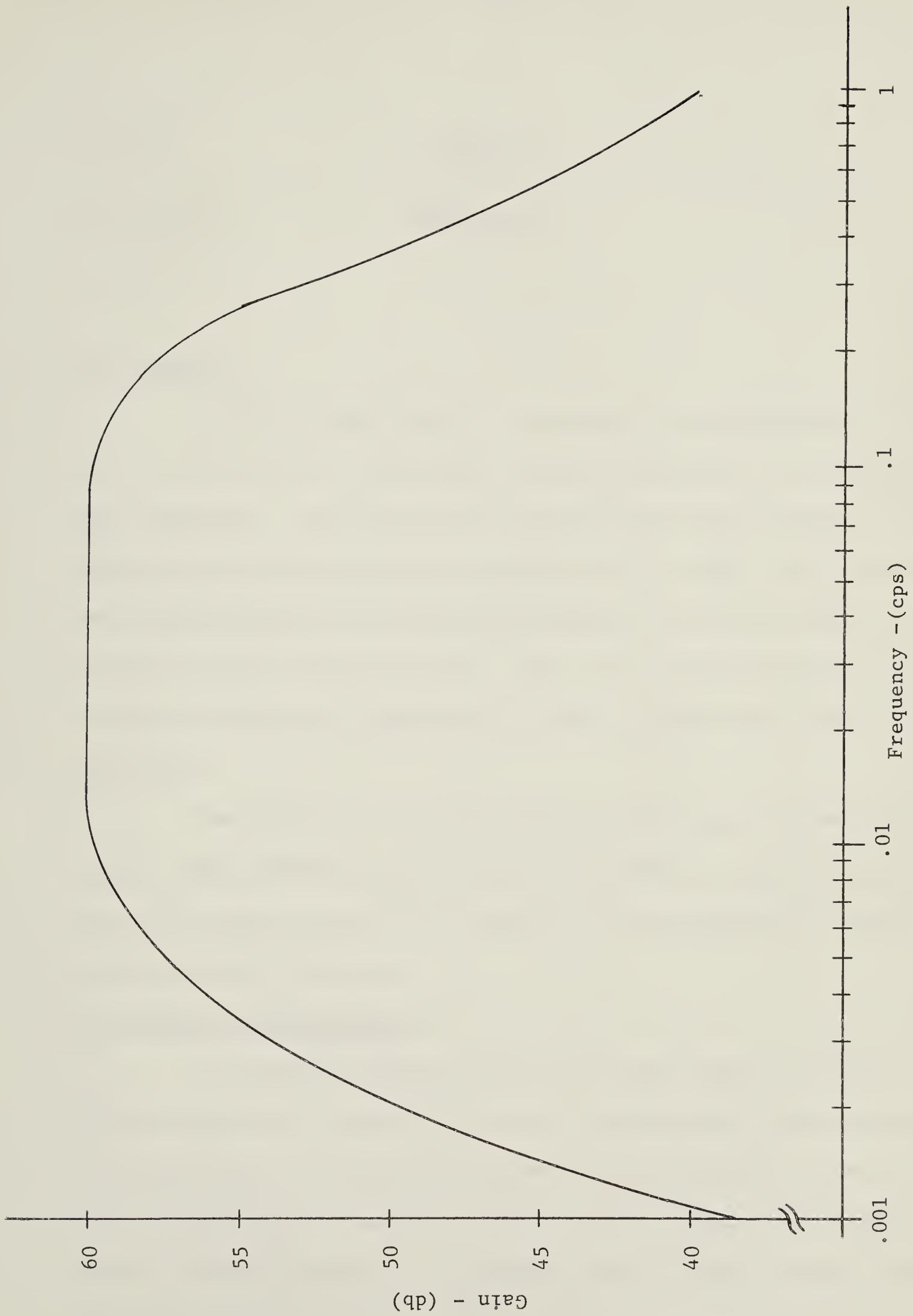
Thus  $A_{vC} = .9968$  and the overall gain is:

$$A_v = A_{vC} A_{vE} = -22.75$$

### 3.3 Measured System Performance

Over a half hour period the total noise referred to the input, with a 10 kilohm source impedance, was found to be less than 7 microvolts peak to peak. The midband gain and input impedance were found to be  $1009 \pm 2\%$  and 2 megohms  $\pm 5\%$ . Measured common mode rejection at midband was -70db. The frequency response of the system from .001 cps to 1 cps is shown in Graph 3.2. The -3db points occur at .0046 cps and .22 cps.





Graph 3.1.1 Frequency Response of Complete System



## CHAPTER IV

### POWER SUPPLY

#### 4.1 General

Since the power supply requirements of the amplifier are very light,  $\pm 12$  volts at 5 milliamps, battery operation is both economical and convenient. The problem with the use of batteries however, is the increase in internal impedance as the battery is used. Such impedances can cause undesired coupling between stages, as well as a marked increase in noise from the battery. Also the resulting decreased output voltage of the battery gives rise to shifts in operating level in the amplifiers.

The addition of a good quality voltage regulator would eliminate these problems as well as make it possible to operate the amplifiers from other external D.C. supplies, either regulated or unregulated, as well as from batteries.

#### 4.2 Regulator Requirements

The regulator should draw only a small amount of current for its own operation, so as to help conserve battery life, when used with a battery supply. Its output resistance should be of the order of  $2\Omega$  and it should have a regulation factor of the order of .001. The regulator should be capable of handling a range of input voltage from 13 volts to 20 volts.



### 4.3 Type

The type of regulator chosen was a series regulator of the following form.

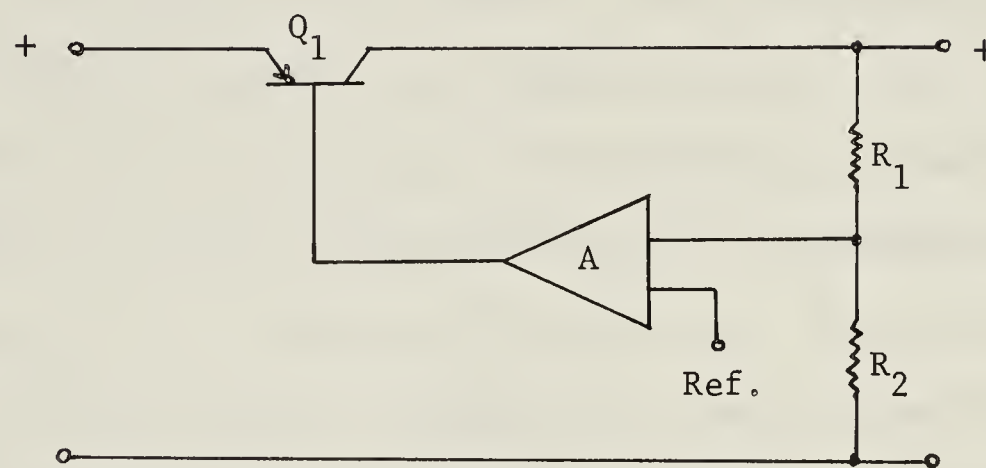


Fig. 4.1 Basic Regulator Circuit

With this circuit regulation is performed by comparing a sample of the output voltage with a reference; any error present is amplified and used to control a series element. The use of the series control element  $Q_1$  in the common emitter configuration rather than the common collector configuration, results in a better regulation factor and lower output resistance.



#### 4.4 Circuit of the +12 Volt Regulator

The circuit of the +12 volt regulator consists of a voltage divider used as a sampler for the output voltage, a zener diode for a reference supply, a differential amplifier to obtain the difference between the reference and the sampled output, and a series control transistor which is operated by the output of the differential amplifier.

Resistor  $R_8$  serves as a starting resistor since the regulator is not of the self-starting type. The action of  $R_8$  can be described as follows. When the input voltage is applied to the regulator the path through  $R_8$ ,  $R_3$  and  $Z_1$  allows  $Q_2$  to be turned on. This sets up sufficient base drive for  $Q_1$  to turn on. During this time  $Q_3$  is still off. The result of  $Q_1$  being turned on is to further increase the voltage at the base of  $Q_2$  and this supplies still more base drive to  $Q_1$  (positive feedback). When the output voltage reaches 12 volts  $Q_3$  turns on and tends to reduce the base drive to  $Q_1$  (negative feedback), and the regulator stabilizes at 12 volts.

$R_8$  should be determined under conditions of the largest load the regulator is required to handle so that starting will always occur for lighter loads. Thus  $R_8$  will act as a crude protection device at the time of starting.

$R_2$  and  $R_4$  are protection resistors. Without these resistors there are several semi-conductor paths from the high side of the power supply to ground with no resistive protection.

The capacitor  $C_1$  across the output serves to suppress transients. Its value was chosen such that the regulator would pass a square wave with very little ringing (slightly underdamped response).



#### 4.5 Circuit of the -12 Volt Regulator

The basic circuit of the -12 volt regulator is the same as that of the +12 volt regulator. Here however the required reference voltage is obtained from the zener diode in the +12 volt regulator. No starting resistor is needed since this circuit is self-starting.

#### 4.6 Regulator Performance

The regulators have been used with both battery input and unregulated D.C. input. Satisfactory results were obtained in all cases.

At the rated load for the +12 volt regulator the output resistance was found to be  $1.95\Omega$  and the regulation factor .0007. For the -12 volt regulator the output resistance was  $2.15\Omega$  and the regulation factor .001.



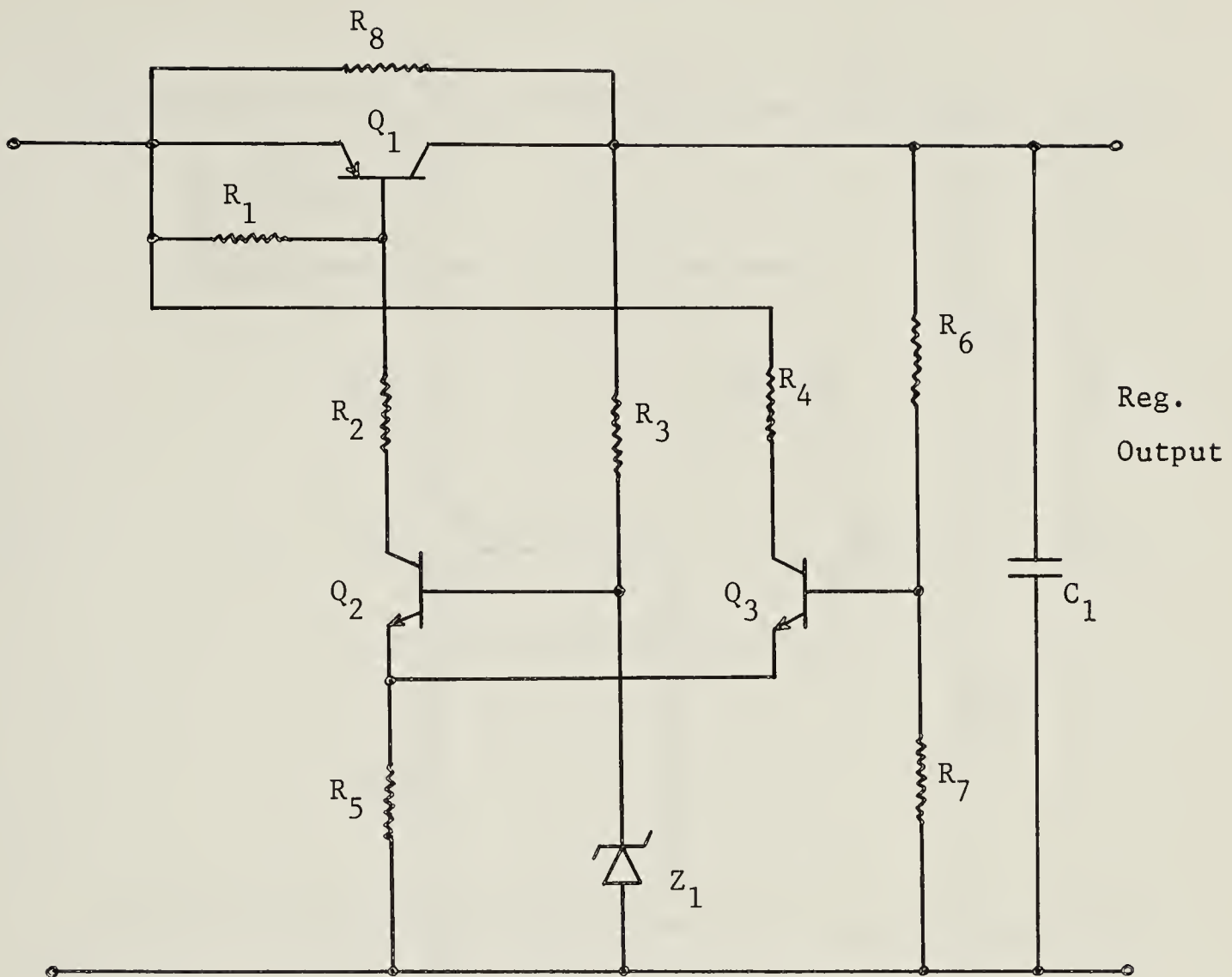


Fig. 4.2 Circuit of +12 Volt Regulator

$$R_1 = 1.5K$$

$$R_2 = 1K$$

$$R_3 = 1K$$

$$R_4 = 1K$$

$$R_5 = 5.6K$$

$$R_6 = 5.6K$$

$$R_7 = 5.6K$$

$$R_8 = 10K$$

$$C_1 = .25\mu f$$

$$Z_1 - 1N752$$

$$Q_1 - 2N3702$$

$$Q_2 - T1415$$

$$Q_3 - T1415$$



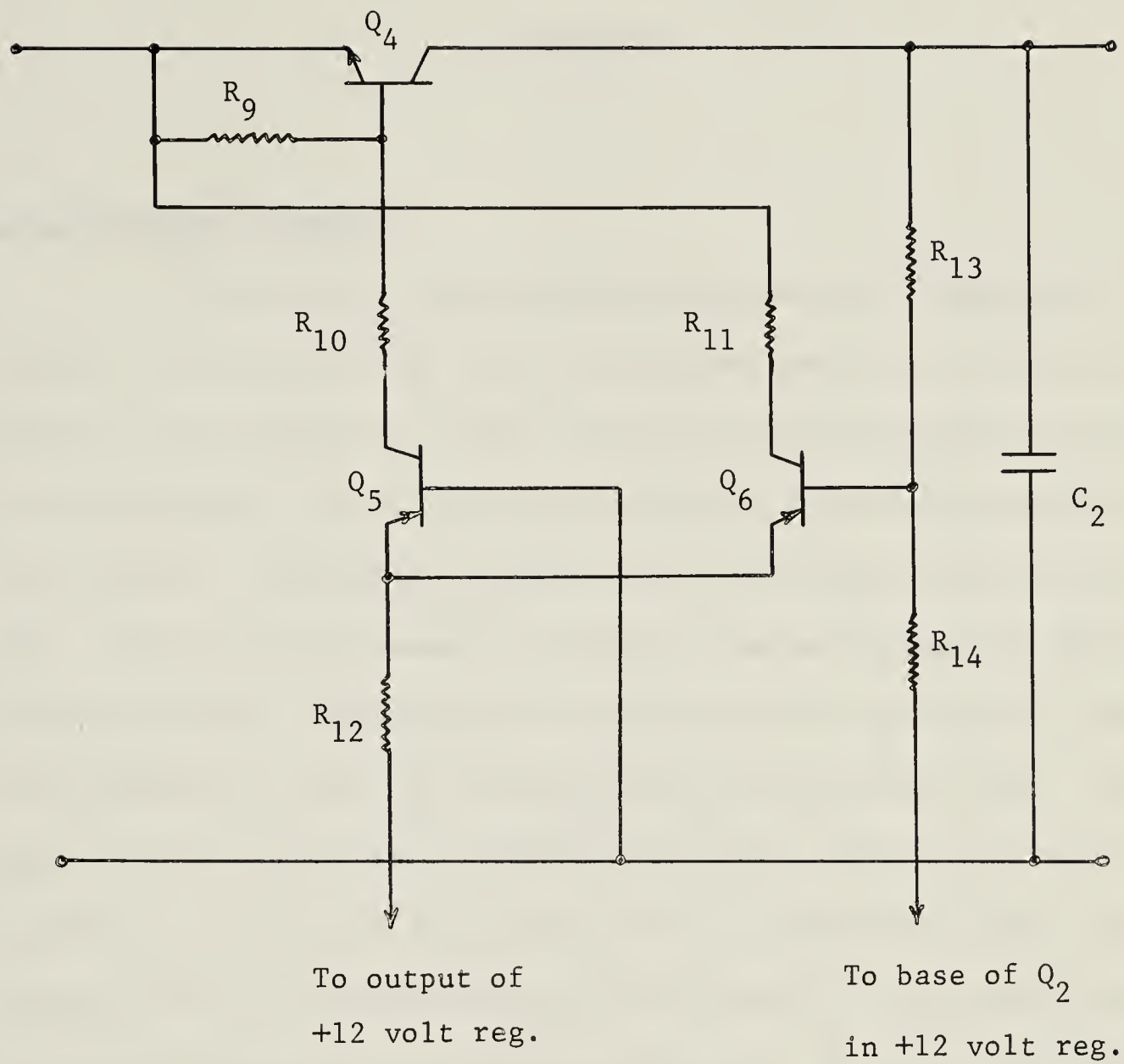


Fig. 4.3 Circuit of -12 Volt Regulator

$$R_9 = 1.5K$$

$$R_{10} = 1K$$

$$R_{11} = 1K$$

$$R_{12} = 12K$$

$$R_{13} = 12K$$

$$R_{14} = 5.6K$$

$$C_2 = .25\mu f$$

$$Q_4 - T1415$$

$$Q_5 - 2N3702$$

$$Q_6 - 2N3702$$



## CHAPTER V

### RECORDING

#### 5.1 Electrode Placement

Both bipolar and unipolar recordings were attempted. In the bipolar recordings the two active electrodes were placed in the lower left portion of the epigastric region and the ground electrode was attached to the right ankle. The active electrodes were placed approximately three inches apart. The results obtained with this bipolar method were quite poor. The recordings seemed to indicate that approximately the same magnitude of signal was appearing at each electrode at the same time, thus the differential action of the amplifier was cancelling them. The best results were obtained with unipolar recording. Here the active electrode was placed in the lower left portion of the epigastric region approximately one inch left of the center line of the abdomen. The passive electrode was placed between the top and the inside part of the right thigh, approximately eight inches above the knee. This position gives minimum interference from the heart since the electrodes are approximately on an isopotential line of the heart. The ground electrode was placed below the right knee. One problem which appeared quite frequently was respiration artifact; however this could usually be eliminated by a slight change in position of the active electrode. The recordings shown in section 5.3 were obtained by using a unipolar electrode arrangement.



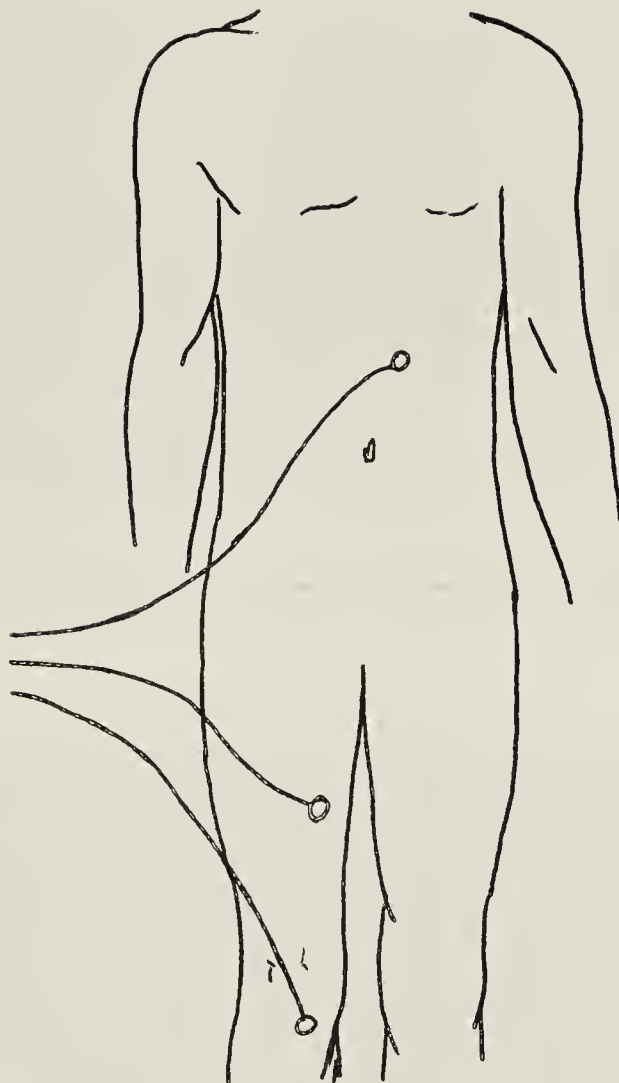


Fig. 5.1 Unipolar Electrode Placement

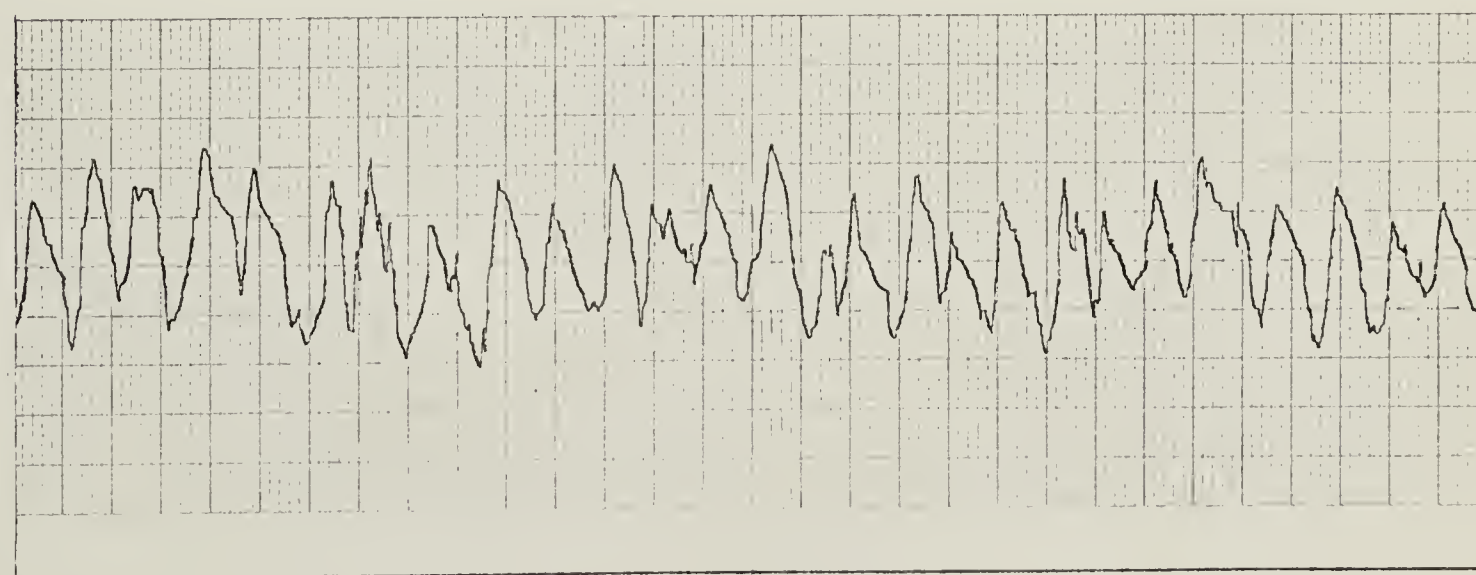
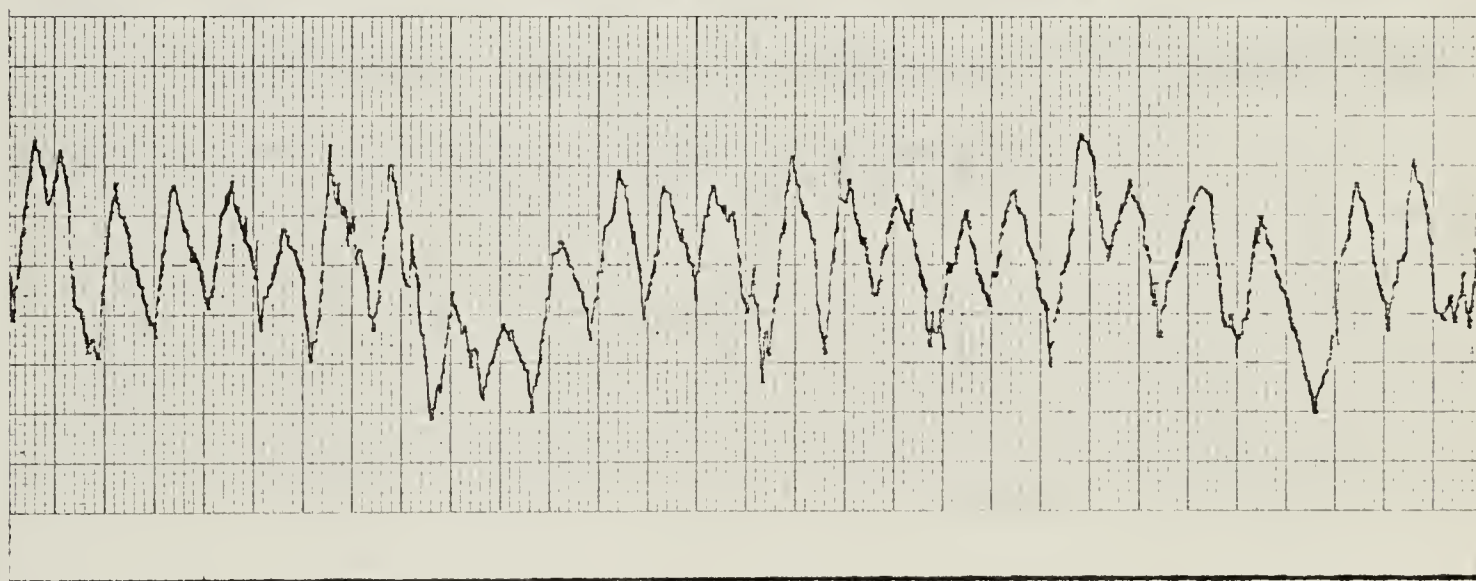
## 5.2 Subject Position

The recordings shown in section 5.3 were all done with the subject lying supine. Although other positions such as sitting, standing and lying on the side were tried, the results were not as good. Better results in these positions could possibly be obtained by choosing different electrode sites.



### 5.3 Examples of Records

Scale: 20 microvolts per mm.  
.25 mm. per second





#### 5.4 Conclusions

At this point a true evaluation of the amplifiers and electrodes is rather difficult, due to the limited amount of recording which has been done. As recording technique is improved, it may be desirable to make certain changes in the instrumentation, however a reasonable amount of success can definitely be achieved with the present set-up. The real remaining problems are in developing a good recording technique and in interpretation of results.

A further consideration is whether the records are truly copies of the electrical activity of the stomach and not some other biological potential. All that can be said with regard to this question is that the general shape, frequency and magnitude of the recordings are in agreement with the results of other investigators.



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# APPENDIX I

## Two Transistor Compound

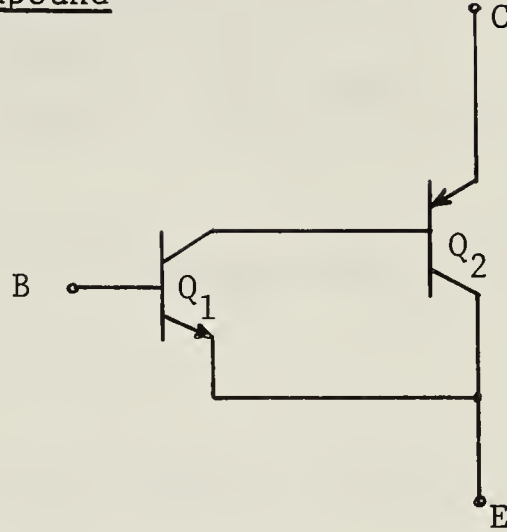


Fig. I-1 Two Transistor Compound

Edwards <sup>(7)</sup> has shown that a compound connected pair of transistors as in Fig. I-1, can be considered as a single NPN transistor with the following h parameters.

$$h_{ie}^* = B \left[ h_{ib1} \frac{1-h_{rb2}}{1+h_{fe2}} + h_{ib2} \left( \frac{h_{ob1} h_{ib1}}{1-h_{rb1}} + \frac{h_{fe1}}{1+h_{fe1}} h_{rb1} \right) \right]$$

$$h_{re}^* = B \left( \frac{h_{ob1} h_{ib1}}{1-h_{rb1}} - \frac{h_{rb1}}{1+h_{fe1}} \right) \left( \frac{1}{1+h_{fe2}} - \frac{h_{ob2} h_{ib2}}{1-h_{rb2}} \right)$$

$$h_{fe}^* = \frac{h_{fe1} (1+h_{fe2})}{1 + \left( \frac{1+h_{fe1}}{1-h_{rb1}} \right) \left( \frac{1+h_{fe2}}{1-h_{rb2}} \right) h_{ob1} h_{ib2}}$$

$$h_{oe}^* = B \left( h_{ob1} + \frac{1-h_{rb1}}{1+h_{fe1}} h_{ob2} \right)$$



$$B = \frac{\left( \frac{1 + h_{fe1}}{1 - h_{rb1}} \right) \left( \frac{1 + h_{fe2}}{1 - h_{rb2}} \right)}{1 + \left( \frac{1 + h_{fe1}}{1 - h_{rb1}} \right) \left( \frac{1 + h_{fe2}}{1 - h_{rb2}} \right) h_{ob1} h_{ib2}}$$

Making the approximations  $h_{fe} \gg 1$ ,  $h_{rb} \ll 1$  and converting to the mixed  $h$  parameters;

$$h_{ib}^* \cong \frac{h_{ib1}}{h_{fe2}} + h_{ib2} (h_{ob1} h_{ib1} + h_{rb1})$$

$$h_{fe}^* \cong \frac{h_{fe1} h_{fe2}}{1 + h_{fe1} h_{fe2} h_{ob1} h_{ib2}}$$

$$h_{rb}^* \cong h_{rb1} + h_{ib1} h_{ob2} + h_{fe}^* h_{ib1} h_{ib2} (h_{ob1})^2$$

$$h_{ob}^* \cong h_{ob1} + \frac{h_{ob2}}{h_{fe1}}$$

Edwards<sup>(7)</sup> has further shown that the addition of the resistor  $R_A$  as in Fig. I-2, reduces the effective current gain of  $Q_2$ .

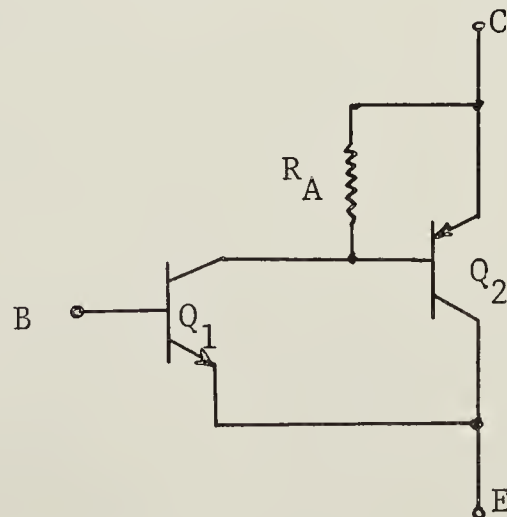


Fig. I-2 Modified Two Transistor Compound



The effective current gain of  $Q_2$  as seen by  $Q_1$  is now;

$$h'_{fe_2} = \frac{1}{\frac{1}{h_{fe_2}} + \frac{1}{\frac{V_{BE_2}}{I_{c_2}} \cdot \frac{I_{c_2}}{I_{c_1}}}}$$

Thus when the resistor  $R_A$  is used,  $h'_{fe_2}$  must be used in place of  $h_{fe_2}$  in the preceding equations.

Rough approximate equations are then;

$$h_{ib}^* \cong \frac{h_{ib_1}}{h'_{fe_2}}$$

$$h_{rb}^* \cong h_{rb_1} + h_{ib_1} h_{ob_2}$$

$$h_{fe}^* \cong h_{fe_1} h'_{fe_2}$$

$$h_{ob}^* \cong h_{ob_1} + \frac{h_{ob_2}}{h_{fe_1}}$$



## APPENDIX II

### Low Pass Active Filter

#### II-1 General

A method is described of synthesizing low pass filters using RC sections within a feedback loop.

The following circuit configuration is used.

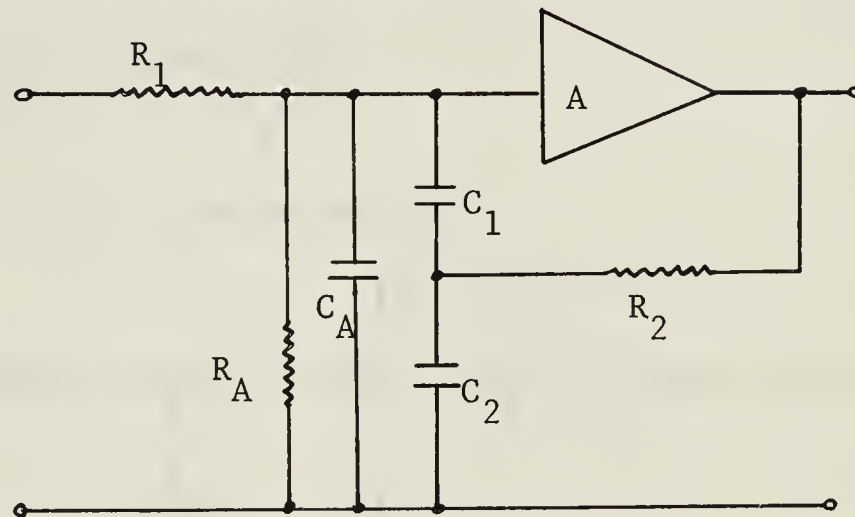
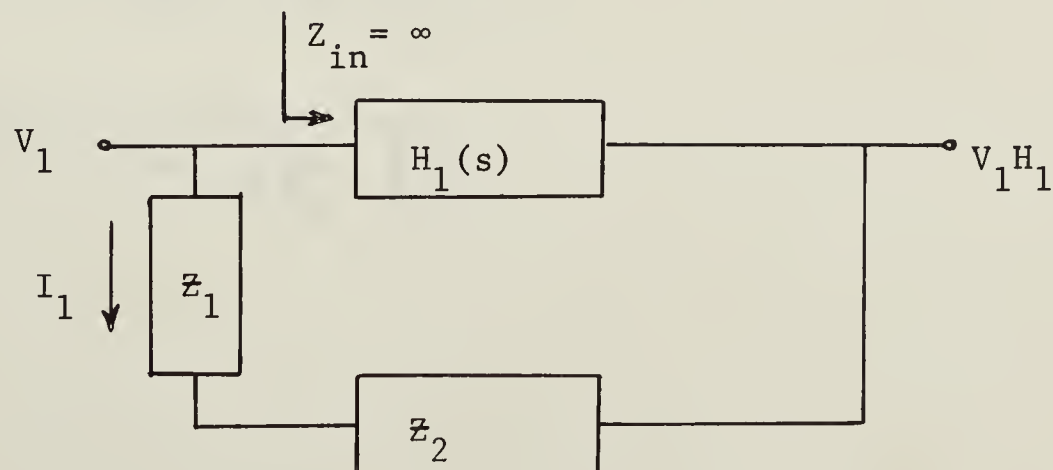


Fig. II-1 Low Pass Active Filter

The amplifier is a D.C. amplifier with an input impedance  $R_A$ . ( $C_A$  can be neglected in this case since  $\frac{1}{R_A C_A} \gg \omega_H$ , where  $\omega_H$  is the high frequency cutoff.

#### II-2 Transfer Function

Assuming  $R_A$  to be infinite the general form of the circuit is:





Where;

$$I_1 = \frac{V_1 (1-H_1(s))}{Z_1 + Z_2}$$

$$Z_B = \frac{V_1}{I_1} = \frac{Z_1 + Z_2}{1 - H_1(s)}$$

In terms of the given circuit;

$$Z_1 = \frac{1}{sC_1}, \quad Z_2 = \frac{R_2}{1 + sR_2C_2}, \quad H_1(s) = \frac{A}{1 + sR_2C_2}$$

$$Z_B = \frac{1 + s(R_2C_2 + R_2C_1)}{s^2R_2C_2C_1 + sC_1(1-A)}$$

The circuit can then be represented as;

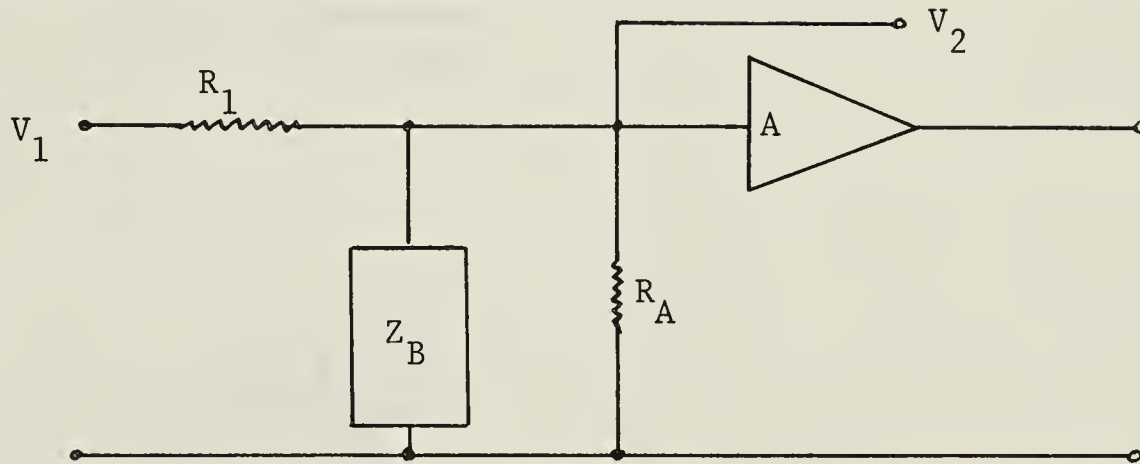


Fig. II-2 Equivalent Circuit

NOTE - The above representation of the given circuit holds only if operation occurs in the linear region of the amplifier.

Defining:

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_B // R_A}{Z_B // R_A + R_1}$$

$$H(s) = \frac{1}{1 + R_1 \left( \frac{1}{Z_B} + \frac{1}{R_A} \right)}$$



$$H(s) = \frac{1}{1 + R_1 \left( \frac{s^2 R_2 C_1 C_2 + s C_1 (1-A)}{1 + s(R_2 C_1 + R_2 C_2)} + \frac{1}{R_A} \right)}$$

Rearranging;

$$H(s) = \frac{\frac{R_A R_2 C_1 + R_A R_2 C_2}{R_1 + R_A} + \frac{R_A}{R_1 + R_A}}{\frac{R_1 R_2 R_A C_1 C_2}{R_1 + R_A} s^2 + \frac{R_A R_2 C_1 + R_A R_2 C_2 + C_1 R_1 R_A (1-A) + R_1 R_2 C_1 + R_1 R_2 C_2}{R_1 + R_A} s + 1}$$

This expression is of the form;

$$H[P] = \frac{2\zeta_1 P + K}{P^2 + 2\zeta P + 1}$$

Where

$$P = T_o s = \frac{s}{\omega_o}$$

$$T_o = \sqrt{\frac{R_A R_1 R_2 C_1 C_2}{R_1 + R_A}}$$

$$2\zeta = \frac{R_A (R_1 C_1 (1-A) + R_2 C_2 + R_2 C_1) + R_1 R_2 C_1 + R_1 R_2 C_2}{\sqrt{R_1^2 R_A R_2 C_1 C_2 + R_A^2 R_2 R_1 C_1 C_2}}$$

$$2\zeta_1 = \frac{R_A R_2 C_1 + R_A R_2 C_2}{\sqrt{R_1^2 R_A R_2 C_1 C_2 + R_A^2 R_2 R_1 C_1 C_2}}$$

$$K = \frac{R_A}{R_1 + R_A}$$



### II-3 Amplitude Response

Letting  $P = ju$

$$H[ju] = \frac{K + 2j\zeta_1 u}{1 - u^2 + 2j\zeta_1 u}$$

$$|H[ju]|^2 = \frac{K^2 + 4\zeta_1^2 u^2}{1 + (4\zeta_1^2 - 2)u^2 + u^4}$$

From this expression the conditions for maximal flatness can be found.

$$2\zeta^2 - 2\zeta_1^2 - 1 = 0$$

$$K = 1$$

NOTE:  $K = \frac{R_A}{R_1 + R_A}$  can never be exactly one. However this condition can be approached very closely and the small amount of error present causes no difficulty. From this point on the assumption will be made that  $R_A \gg R_1$  or that  $K = 1$ .

Defining:  $\gamma = 2\zeta^2 - 2\zeta_1^2 - 1$

$$\text{then } |H[ju]|^2 = \frac{1}{1 + \frac{u^4 + 2\gamma u^2}{4\zeta_1^2 u^2 + 1}}$$

Defining:  $x = \frac{u^4 + 2\gamma u^2}{4\zeta_1^2 u^2 + 1}$

$$\text{then } |H[ju]|^2 = \frac{1}{1+x}$$

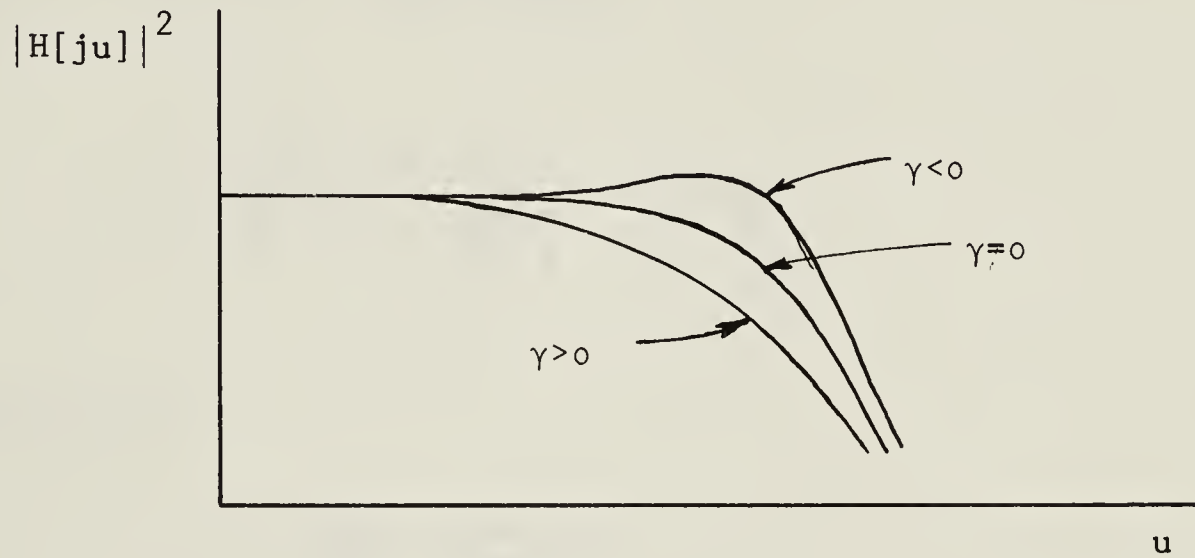


For  $\gamma > 0$  then  $x > 0$  for all values of  $u$ , and thus  $|H[ju]|^2 \leq 1$  for all values of  $u$ .

For  $\gamma < 0$  then for certain values of  $u$ ,  $x < 0$  and thus a boost will exist.

For  $\gamma = 0$  the system is maximally flat.

Thus different conditions at roll-off may be obtained by adjustment of  $\gamma$ .



#### II-4 Design Equations

The design equations considered here will be only for the maximally flat condition.

The amplitude response is then:

$$|H[ju]|^2 = \frac{1}{1 + \frac{u^4}{1 + 4\zeta_1^2 u^2}}$$

$$T_o^2 = \frac{R_A R_1 R_2 C_1 C_2}{R_1 + R_A}$$

with  $R_A \gg R_1$

$$T_o^2 \cong R_1 R_2 C_1 C_2$$



$$\frac{\zeta}{\zeta_1} = \frac{R_A(R_2C_1 + R_2C_2 + R_1C_1(1-A) + R_1(R_2C_1 + R_2C_2))}{R_A(R_2C_1 + R_2C_2)}$$

With  $R_A \gg R_1$

$$\frac{\zeta}{\zeta_1} \cong 1 + \frac{R_1C_1(1-A)}{R_2C_1 + R_2C_2}$$

$$\zeta_1 \cong \sqrt{\frac{R_2}{R_1}} \frac{C_1 + C_2}{2\sqrt{C_1C_2}} \frac{1}{\sqrt{1 + \frac{R_1}{R_A}}}$$

with  $R_A \gg R_1$

$$\zeta_1 \cong \sqrt{\frac{R_2}{R_1}} \frac{C_1 + C_2}{2\sqrt{C_1C_2}}$$

In most cases there is no advantage to not making  $C_1 = C_2$ .

This results in further simplification of the design equations.

$$T_o^2 \cong R_1R_2C^2$$

$$\frac{\zeta}{\zeta_1} \cong 1 + \frac{R_1(1-A)}{2R_2}$$

$$\zeta_1 \cong \sqrt{\frac{R_2}{R_1}}$$

NOTE:  $\zeta_1$  has the same effect on the roll-off in this low pass filter as it has on the roll-off in the bootstrapped bias input.



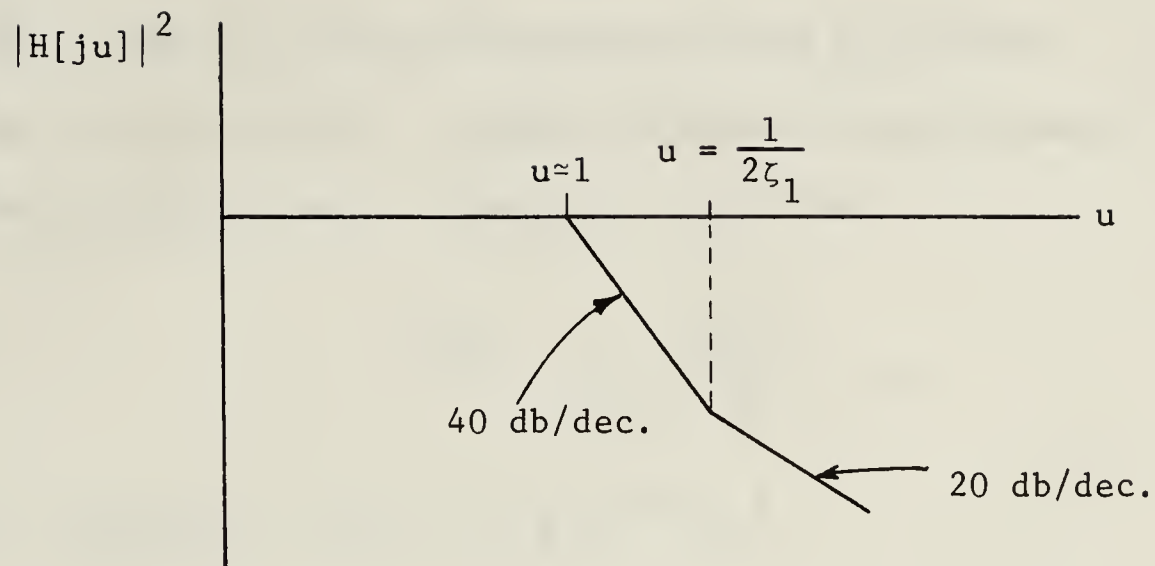


Fig. II-3 Roll-off of Low Pass Filter With Small  $\zeta_1$ .

As  $\zeta_1$  increases the frequency range covered by the 40db per decade roll-off decreases. Thus for best attenuation  $\zeta_1$  should be made as small as possible.



## ERRATA

- (1) Page 2, Fig. 1.1 the word Phylorus should be Pylorus.
- (2) Page 10 should read:  $\omega_o$  being the normalizing frequency.
- (3) Page 27 should read: Letting  $C_1 = C_2 = 20\text{uf}$

Then 
$$\zeta_1 = \sqrt{\frac{R_2}{R_1}} = .25$$

$$T_o^2 = R_1 R_2 C_1 C_2 = .722 = .25 \times 10^{-10} R_1^2$$

and  $R_1 = 170\text{K}$

$$R_2 = 10.6\text{K}$$

- (4) Page 28

$$R_1 = 180\text{K}$$

$$R_2 = 10\text{K}$$

$$C_1 = 20\text{uf}$$

$$C_2 = 20\text{uf}$$





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